

Empirical Equations (4.8.3)

‘relying upon or derived from observation or experiment’

Different from actual physical laws

Standard approach to developing empirical equations is by regression analysis

Steps:

1. Select variables of interest: dependent variable (Y) and independent variables or predictor variables (X_1, X_2, \dots, X_p)
2. Formulate regression model, linear additive model.

$$\hat{Y} = b_0 + b_1 * X_1 + b_2 * X_2 + \dots + b_p * X_p$$

both $b_0 \dots b_p$ are regression coefficients
and $b_0 = \textit{regression constant}$

2 (continued)

In hydraulics most common model is the linear multiplicative model

$$\hat{Y} = c_0 * X_1^{c_1} * X_2^{c_2} * \dots * X_P^{c_P}$$

But either form doesn't matter because we can put the multiplicative model in additive form by log transforming the multiplicative model

$$\widehat{\log Y} = \log c_0 + c_1 * \log X_1 + c_2 * \log X_2 + \dots + c_P * \log X_P$$

The hat denotes the estimate of the average value and leads to uncertainty

- 1) model is imperfect
- 2) coefficients are derived for a specific set of data

3. Collect your data.

N measured values, for variables which must be associated in space and time

4. Calculate values of coefficients

R or SAS really make life easier (sometimes)

$$\sum_{i=1}^N (y_i - \hat{y}_i)^2 \quad \text{or} \quad \sum_{i=1}^N (\log y_i - \widehat{\log y_i})^2$$

where y_i is the actual measured values and \hat{y}_i are the values estimated by the regression equation

Why is the regression so different from the physical laws?

- 1) variables are chosen by us
- 2) we determine what the equation looks like
- 3) coefficients are determined by measured data
- 4) association is not causation

Lastly, regression equations do not transfer across systems

Empirical equations and dimensional analysis

Very similar application as regression

except we apply the regression to the dimensionless quantities

$$\widehat{\Pi Y} = c_0 * \Pi_1^{c1} * \Pi_2^{c2} * \dots * \Pi_P^{cP}$$

Where Π is the dimensionless term.

Great part is that whatever regression model is used, we don't have to worry about changing the equation to deal with different unit systems

Discussion of the application for predicting velocity

There is still a lot of scatter around the line. To reduce scatter

- 1) account for effects of other terms
- 2) look for other factors not included in regression